# Mark Iwen

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## BIO SKETCH

Mark Iwen joined the MSU faculty in the fall of 2013, and holds joint appointments in the Department of Computational Mathematics, Science, and Engineering, and the Department of Mathematics.

Prior to arriving at MSU, Dr. Iwen was a visiting assistant professor in the Department of Mathematics at Duke University (September 2010–August 2013) and had been a Postdoctoral Fellow at the Institute for Mathematics and its Applications (IMA), at the University of Minnesota (September 2008–August 2010). Dr. Iwen received his PhD in applied and interdisciplinary mathematics from the University of Michigan in 2008, and a B.S. in computer science and mathematics from the University of Wisconsin, Milwaukee, in 2002.

## RESEARCH INTERESTS

Signal processing, computational harmonic analysis, algorithms for the analysis of large and high-dimensional data sets

## WEBSITE

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## CURRENT RESEARCH FOCUS

Many signal processing applications, such as image, video, and music compression, deal with finding compact representations of signals (e.g., generating compressed image files, music files, etc.). Compactly representing such signals is beneficial for many reasons. Compressed signal files are faster and cheaper to communicate and store. In more extreme situations, data signals may be so large, or change so rapidly, that they can not be stored or analyzed at all without first being quickly compressed. Many interesting problems of this type exist in research areas related to algorithms for massive datasets (e.g., Internet data analysis), and wideband analog-to-digital conversion. In other settings, signal data may be small enough to store in memory without compression, but still be too voluminous to allow it to be analyzed in a reasonable amount of time. In these cases it may be desirable to quickly compress the signal data to a more manageable size, analyze it in its compressed form, and then recover approximate characteristics of the original data. Many useful statistical methods and techniques from numerical linear algebra fit this general description. My research principally falls within the intersection of computational mathematics, harmonic analysis, and signal processing, with an emphasis on compression-based approaches akin to those mentioned above. As a result, my research products often take the form of efficient computational methods, supported by rigorous theoretical guarantees, for the analysis (or approximation) of large and high-dimensional data sets. Two examples follow.

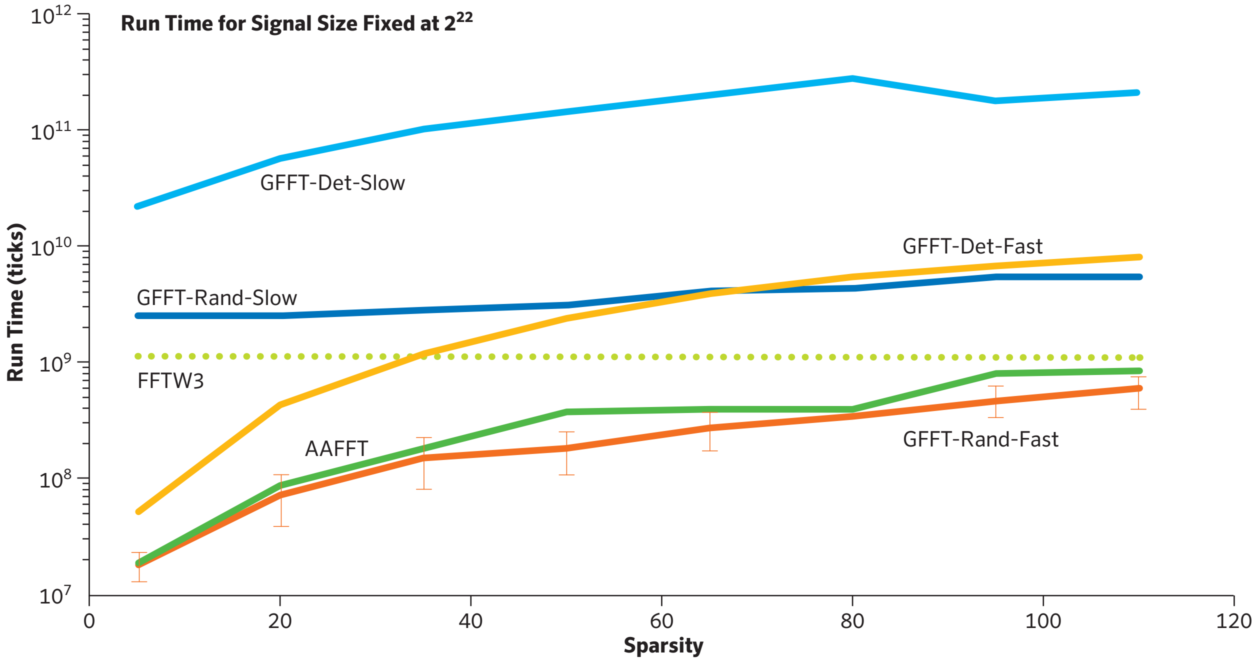


**FIGURE 1.** Run time of several SFTs compared with a standard FFT (FFTW) for frequency-sparse functions. The vertical axis is run time. The horizontal axis is the number of sine and cosine functions needed in order to accurately approximate the tested frequency-sparse functions. The absolute value of all frequencies is bounded above by $2^{22}$. The run time of FFTW is graphed as the horizontal dotted green line. The fastest SFT implementation was introduced in the paper immediately above (i.e., see the orange line).

***Sparse Fourier transforms.*** The Discrete Fourier Transform (DFT) is utilized in a tremendous number of signal processing and computational settings. The naive algorithm for computing the DFT of a vector $\mathbf{v} \in \mathbb{C}^N$ uses $O(N^2)$ operations. In the 1960s Cooley and Tukey introduced the Fast Fourier Transform (FFT) , which can compute the DFT using only $O(N \log N)$ operations. This reduction in computational complexity has had such far reaching effects that the FFT has been widely lauded as one of the ten most important algorithmic developments of the twentieth century.

Developed more recently, *sparse Fourier transforms (SFTs)* compute an approximate, or compressed, version of a vector's DFT using only a number of operations proportional to the Fourier sparsity of the spectrum of the signal (i.e., the number of dominant frequencies), as opposed to the length of the signal, $N$. This makes SFTs faster than the FFT for vectors whose DFTs contain a relatively small number of large Fourier coefficients at unknown frequencies. More specifically, SFTs use only a small subset of the input data and run in time proportional to the sparsity or desired compression, considerably faster than in time proportional to the signal length. This is made possible by requiring that the algorithms only report the dominant Fourier coefficients and frequencies, rather than all Fourier coefficients.

Over the past several years I have developed several SFT methods, with accompanying theoretical guarantees, for computing near-optimal sparse approximations to the sequence of Fourier coefficients of a given periodic $C^1$-function, $f : [0, 2\pi]^D \to \mathbb{C}$. These algorithms use a number of operations proportional to a given value $k$ in order to find and output the $k$ Fourier coefficients of $f$ whose magnitudes are largest. Possibly most interesting among these results was the development of the first, and so far only, entirely deterministic algorithm which achieves the same type of mixed- norm error guarantees as the (asymptotically) exponentially slower Fourier compressive sensing techniques.

There are many interesting and difficult problems to pursue related SFTs in the future, including: improving their theoretical performance guarantees for the approximation of functions of many variables (current operation counts for methods with near-optimal error guarantees scale sub-optimally in the dimension, $D$, of the function's domain), and producing more efficient and user friendly implementations. It is also interesting to consider extending these sparse approximation algorithms to other orthonormal basis functions. Along these lines, I am currently producing similarly fast sparse transforms for computing the largest Legendre coefficients of a given analytic function.

***Near-optimal ΣΔ quantization.*** The process of quantizing analog signals into binary form is fundamental to signal processing. In perfect conditions this process is straightforward: a real number $v \in [0, 1]$ can be quantized by testing whether it is greater than ½ or not, belongs to [¼, ¾) or not, etc.. A series of $r$ such tests leads to the first $r$ digits of $v$ in binary, and the resulting quantization error is thus $2^{-r}$. A unit vector $\mathbf{v} \in [0, 1]^N$ can be quantized component-wise in an analogous fashion, leading to a quantization error $\sim C \cdot 2^{-r/N}$ , for a fixed $C \in \mathbb{R}^+$, whenever $r$ bits are used.

Unfortunately, this simple binary encoding technique is not robust to quantization mistakes under less perfect conditions: just a few mistakes while encoding the most significant bits of even a single entry in $\mathbf{v}$ can lead to large errors. More often-used ΣΔ-quantizers, on the other hand, produce an alternate bit string encoding of a given vector $\mathbf{v}$ which is far more robust to arbitrary bit mistakes made during quantization. Recently, Rayan Saab and I designed the first known family of ΣΔ-schemes whose quantization error is guaranteed to decay exponentially in the number of bits, $r$, used in order to quantize any given unit vector, $\mathbf{v} \in \mathbb{R}^N$, (i.e., the error is $\sim 2^{-cr/N}$ for a fixed $c \in \mathbb{R}^+$). Thus, their quantization errors decay near-optimally despite the fact that they are more robust to arbitrary errors made during quantization than more naive schemes are. In the future we plan to expand this family of quantization schemes in order to include a larger number of the ΣΔ-quantizers currently in common use.

## RECENT PUBLICATIONS


M.A. Iwen, F. Krahmer, "Fast Subspace Approximation via Greedy Least-Squares," *Constructive Approximation*, vol. 42 no. 2, pp. 281–301 (2015).

M. Iwen, A. Viswanathan, Y. Wang, "Robust sparse phase retrieval made easy," *Applied and Computational Harmonic Analysis* (2015).

M.A. Iwen, "Compressed sensing with sparse binary matrices: Instance optimal error guarantees in near-optimal time," *Journal of Complexity*. 2014;30(1):1–15 (2014).

A.C. Gilbert, P. Indyk, M. Iwen, L. Schmidt, "Recent developments in the sparse fourier transform: A compressed fourier transform for big data," IEEE *Signal Processing Magazine*, 31(5):91–100 (2014 ).